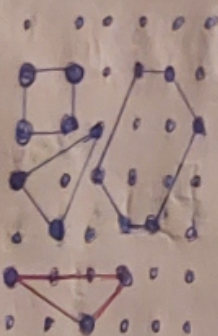


Nrich maths problem

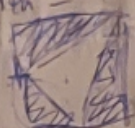
Pick's Theorem



To check the enclosed areas, put shapes into squares.

1. Area = $1 \times 1 = 1$ ✓

2. Area =



Not shaded = shape
Total area = $2 \times 2 = 4$
Shape = $4 - \frac{1}{2} - 1 - 1$
= $1 \frac{1}{2}$ ✓

3.



Not shaded = shape
Total area = $1 \times 3 = 3$
Shape = $3 - \frac{1}{2} - 1$
= $1 \frac{1}{2}$ ✓

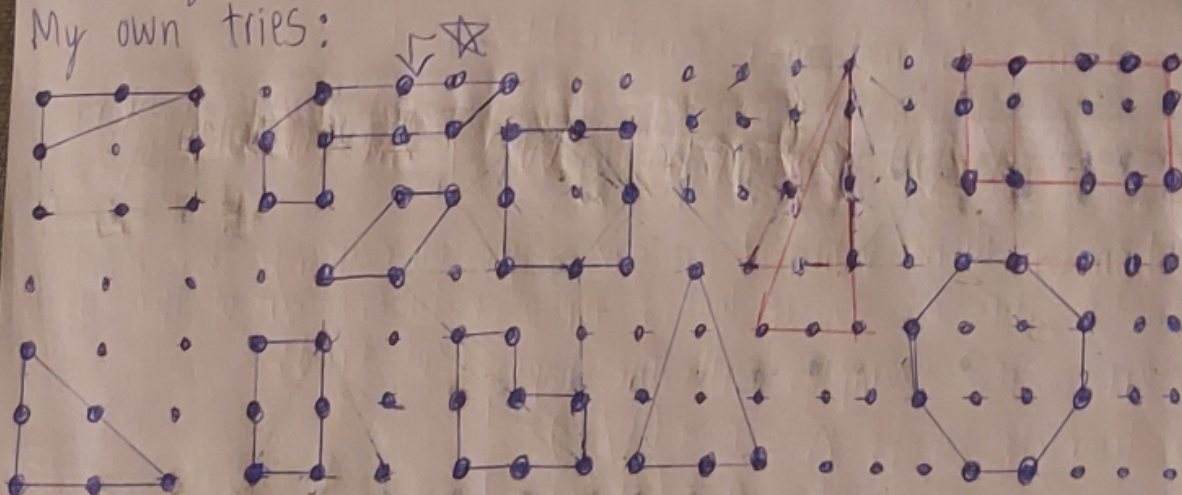
4.



Not shaded = Shape
Total area = 9

1. Square = (p,i) of (4,0) and enclosed area of 1.
2. Blue triangle = (p,i) of (3,1) and enclosed area of $1 \frac{1}{2}$.
3. Red triangle = (p,i) of (5,0) and enclosed area of $1 \frac{1}{2}$.
4. Blue hexagon = (p,i) of (6,4) and enclosed area of 6.

My own tries:



- Blue triangle = (p,i) of (4,0) and enclosed area of 1.] 2 figures can be described as (4,0)
- Parallelogram = (p,i) of (4,0) and enclosed area of 1.] They all have an area of
- Right angled triangle = (p,i) of (6,0) and enclosed area of 2] 2 figures with (6,0)
- Rectangle = (p,i) of (6,0) and enclosed area of 2.] All have an area of 2
- Isosceles triangle = (p,i) of (4,2) and enclosed area of 3.] 2 figures with the same area of 3
- Blue square = (p,i) of (8,1) and enclosed area of 4.] * increase sum of p and i
- Red right-angle triangle = (p,i) of (7,0) and enclosed area of 4.] * increase area by 1

* There are many ways such as adding one more dot inside and outside of the shape (to the L) and adding 3 to the shape but non-
inside (triangle).

I've noticed that even though 2 different shapes had the same ^{total between} P and A , they don't give the same area. It doesn't happen when you increase the P by 1, to 4. Because some are the same but I didn't find them. To increase the area by one you always have to add the space of a square (like the one in the example). All ways lead to this in one way or another.

Hexagon = (P, i) of $(8, 4)$ and enclosed area of 7.

Red rectangle = (P, i) of $(12, 3)$ and enclosed area of 8.

3-d folded paper shape $\star$ = (P, i) of $(10, 0)$ and enclosed area of 4.

All The areas I calculated using the method in The first page with the examples

Can you find a relationship between the variables?

Yes, there is a relationship. At first I didn't see it but little by little I got nearer results until I tried with all of them and it worked. The relationship I found was:

$$\text{Enclosed area} = \text{Internal dots} + (\text{Perimeter dots} / 2) - 1$$

* Full explanation on WHY the formula is like this

To find one of them when 2 are given, you can rearrange the formula to make the one you're looking for alone in one part of the equation.

Here are a few examples with the shapes I've tried:

Blue triangle $(4, 0)$: $A = I + (P/2) - 1$

$$A = 0 + (4/2) - 1$$

$$= 0 + 2 - 1$$

$$= 1 \text{ sq} \checkmark$$

Blue square $(8, 1)$: $A = I + (P/2) - 1$

$$A = 1 + (8/2) - 1$$

$$= 4 \text{ sq} \checkmark$$

Red rectangle $(12, 3)$: $A = I + (P/2) - 1$

$$A = 3 + (12/2) - 1$$

$$= 2 + (12/2) \checkmark$$

$$= 9 + 6 = 8 \text{ sq} \checkmark$$

Can you find the area of any polygon once you know the number of perimeter points and interior points?

Yes you can. You can substitute the values by the letters of P and I, P being the perimeter points and I being the interior points. so with the formula:

$$A = I + (P/2) - 1$$

you can calculate the area by dividing the perimeter points by 2, taking away 1 and adding the interior points. Lets try this with an example of a polygon. Lets say its p, i is (9, 4). so to calculate the area we:

$$A = I + (P/2) - 1$$

$$A = 4 + (9/2) - 1$$

$$= 4 + 4.5 - 1$$

$$= 4 + 3.5$$

$$= 7.5 \text{ square units}$$

We know this is correct because as show above I have tried this with some of my examples and it gives the same area as I had calculated before with my method. Here is even more reliable proof of it by using the area given for the blue hexagon in the diagram provided by nrich:

$$\text{Area} = 6 \quad (p, i) = (6, 4)$$

$$A = I + (P/2) - 1$$

$$6 = 4 + (6/2) - 1$$

$$= 4 + 3 - 1$$

$$= 7 - 1$$

$$= 6$$

$$6 = 6?$$

✓
* 1. Internal dots: They are fully inside the polygon so are counted fully. They are needed in this formula as they tell us the polygon's shape more precisely and the amount of space inside it.

2. The perimeter points are needed to know the length of the polygon.

3. You divide it by 2 as the points are in both sides so they only count once.

4. The -1 at the end is a correction factor as the first point is counted twice, once for each of its adjacent sides.

So to solve this I first just added P and I but it didn't make the area. Then I thought about it and saw that half of the dot was inside and half outside so I divided the number of dots by 2 so only the part inside the shape count. Then I saw ~~try my~~ some of my examples that all of them were more than the real area so I -1 to all and created the formula.