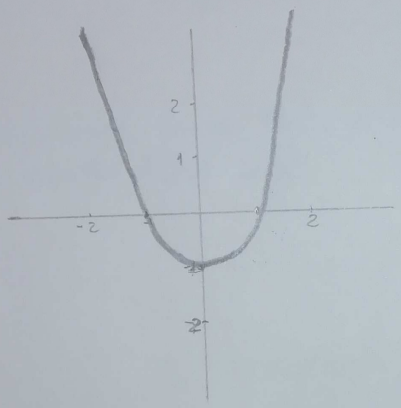
Curve Hunter

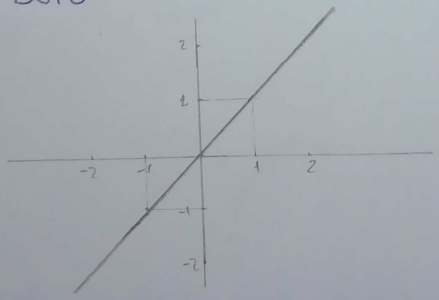
Sketch possible curves that satisfy the following conditions:

1- A continuous curve with exactly one point with zero gradient and exactly two zeros.



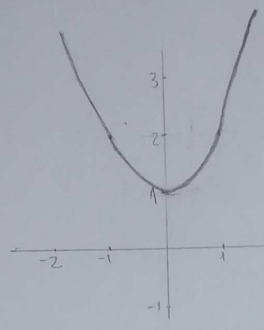
$f(x) = x^2 - 1$

2- A continuous curve with no points with zero gradient and exactly one zero



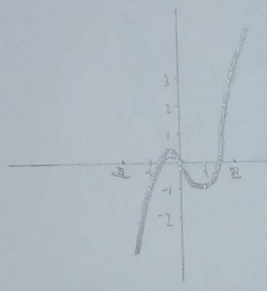
$f(x) = x$

3- A continuous curve with exactly one point with zero gradient and no zeros.



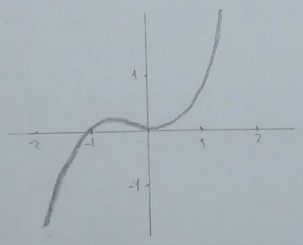
$f(x) = x^2 + 1$

4- A continuous curve with exactly two points with zero gradient and exactly three zeros.



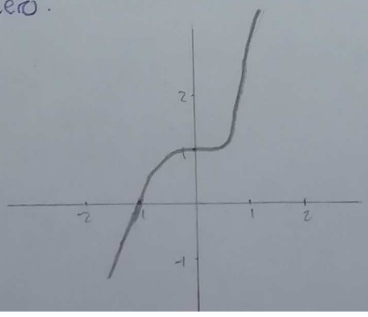
$f(x) = x^3 - x^2 - x$

5 - A continuous curve with exactly two points with zero gradient and exactly two zeros



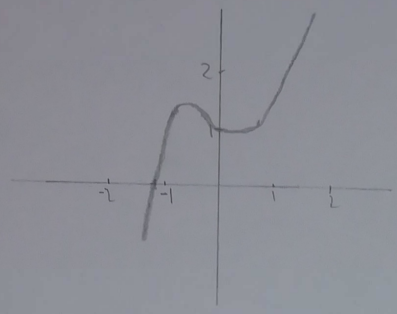
$f(x) = x^3 + x^2$

6- A continuous curve with exactly one point with zero gradient and exactly one zero.



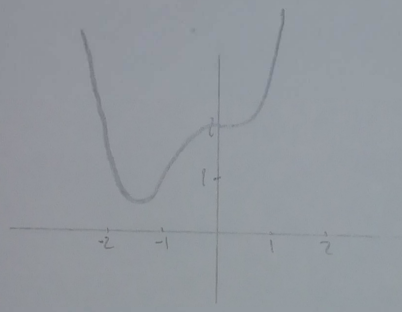
$f(x) = x^3 + 1$

7- A continuous curve with exactly two points with zero gradient and exactly one zero.



$F(x) = X^5 + X^2 + 1$

8- A continuous curve with exactly two points with zero gradient and no zeros.



$F(x) = X^4 + 2X^3 + 2$

Is there a number of zeros and a number of points with zero gradient for which a continuous curve cannot be drawn? Or is it possible to draw a continuous curve with any number of zeros and any number of points with zero gradient?

Continuous curves can be drawn when:

a = number of points with zero gradient

b = number of zeros.

$a \in \mathbb{N}_0$

$b \in \mathbb{N}_0$

+ When "a" is 0: $b = 1$

+ When "a" is a number other than 0:

$$\{b \in \mathbb{N}_0 / 0 \leq b \leq a+1\}$$