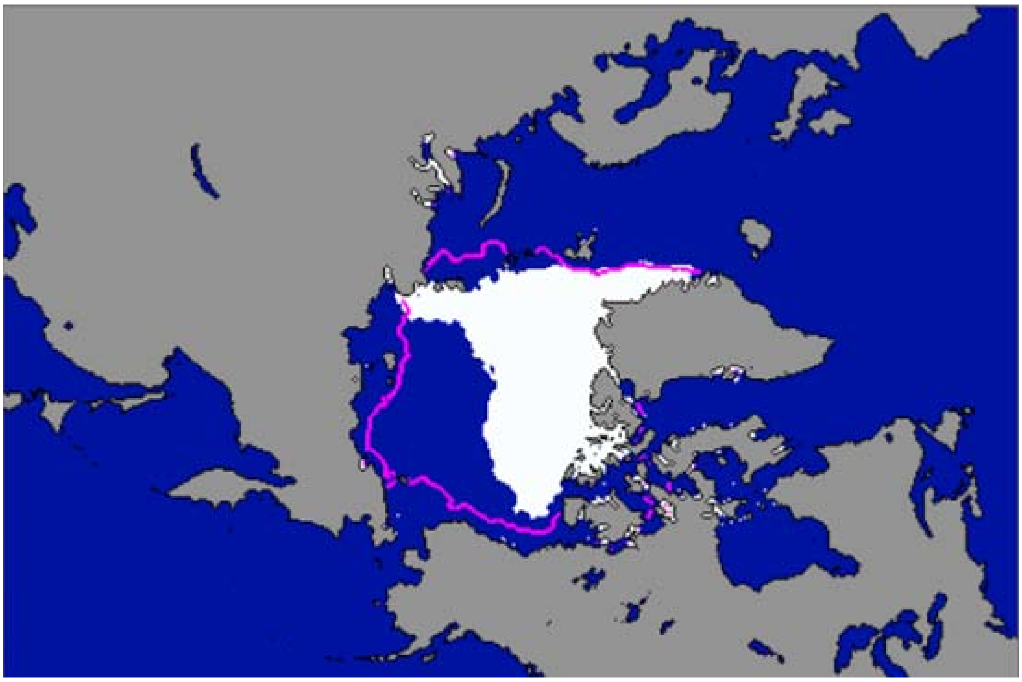
ARCTIC SUPER MODELS

The Arctic ice cap is in trouble. Due to global warming, summer sea ice cover has been shrinking by an area the size of Scotland every year. Measurements from submarines indicate that the ice has grown thinner by at least 40% over the last two decades. Predictions of if and when the permanent ice will disappear from the Arctic vary widely, but few scientists give it longer than 100 years and many predict that a total melt-down of the Arctic will occur within our lifetimes.

"All our predictions about the future of the Artic ice cap come from sophisticated mathematical models."

Peter Wadhams, Professor of Ocean Physics, University of Cambridge

Image courtesy National Snow and Ice Data Center



Sea ice extent in September 2007. The pink line indicates the average extent over the years 1979 to 2000.

But how do scientists make these predictions? A lot depends on data like that currently being gathered by the Catlin Arctic Survey. The ice team are measuring the thickness of the ice, and knowing how thick the ice is now is crucial in understanding how it will change in the future. Scientists will feed these measurements into what are known as climate models or sea ice models. These models are based on mathematics and physics, and they provide a much-needed glimpse into the future of the Arctic and the impacts of climate change.

Put simply, a mathematical model is a description of reality using equations and mathematical formulae. For example, imagine you are interested in the population of polar bears living in the Arctic. You may have observed that over the last few years, their number has halved every year. You could then use the mathematical equation:

$$y = {}^{1}/_{2}\, x$$

to describe the decline. Here x is the number of polar bears in a given year and y the number of polar bears in the following year. You can then use the equation to 'predict the future': if there are 100 bears today, then the equation tells us that there will be 50 next year, 25 the year after, and so on.

The world's favourite Arctic inhabitant

This particular model is pretty crude. For example, it doesn't take account of the fact that the population of polar bears depends on the number of ringed seals there are for them to eat. Their present decline might be due to a decline in the number of seals, and stop once the seal population has stabilised. This fact should really be included in a model – but once you start thinking about it, there is almost no end to the factors you might include. The seals themselves depend on the availability of cod, their staple diet. And what about water temperature which affects the cod population? Or human interventions like fishing?

The trick in building a mathematical model is to start simple: think carefully about the situation you're describing, and concentrate on just one or two of the most important factors. In our example, let's stick to the number of ringed seals. You then look at all the information you have, the sizes of the ringed seal and polar bear populations over the last few years, and see if you can spot a pattern. For example, you might spot that the number of polar bears seems to decline when there are less than 1000 seals and increase when the number of seals grows to more than 1000. Your model could then be improved to become:

$$y = {}^{s}/_{1000}\, x,$$

where s is the number of seals. If s is greater than 1000, then $s/1000$ is greater than 1, so the polar bear population grows from year to year. If s is less than 1000, then $s/1000$ is less than 1, so the polar bear population decreases every year. This model reflects the dependence of polar bears on ringed seals.

image © Greenpeace

A ringed seal, the polar bear's favourite food.

Once you have a model you're happy with, you go back and compare it to reality. Does it match the information you have about seal and bear populations? Does it accurately predict the population size based on past bear numbers? If it does, you can assume that seal numbers really are the most important factor impacting on bear population, so you can protect the bears by protecting the seals, making sure their number remains above 1000. You can also include your model in other, more complicated, models on the whole Arctic ecosystem. If your model does not predict real data accurately, you need to introduce another layer of complexity, for example you might include the cod population and water temperature. The models used by scientists to predict animal populations are more complex than our example here, but are built up in a similar way.

As you can see, building mathematical models amounts to a subtle interplay of mathematics and observation. You observe reality, try and capture the patterns you spot using mathematics, and then go back to see if your formula really matches reality. You keep improving your model until you feel it is sufficiently accurate for your purpose.

Some mathematical models are so good that we have come to accept them as laws of nature. For example, when you give a ball a push, then the resulting acceleration of the ball is equal to the strength of the push (the force applied) divided by its mass. This rule, known as Newton's second law of motion, is so accurate that we have almost forgotten that it is really a mathematical model of reality, given by the equation:

$$F=ma$$

where F is force, a is acceleration and m is mass. We feel that this law is reality, rather than just an incredibly accurate description of reality.

Most models, and especially climate models, are far more complicated than this, consisting of many different components. In climate modelling, scientists consider equations describing the energy from the Sun and how it interacts with the Earth. For example, ocean currents like the Gulf stream can transport water that has been heated in one part of the globe to another, warming the coasts of land that would otherwise be cooler. The ice at the Poles reflects a much larger amount of the Sun's energy than open water does, so when ice cover is reduced, more energy is absorbed by the Earth and this increases the rate of global warming.

Complex mathematical models are used to predict the Earth's climate

All these factors are combined into complex large-scale models, which give the predictions you see in the newspaper headlines. Most of the time the physics and maths doesn't actually make it into these headlines, but without them no predictions would be possible.

To find out more about modelling climate change, have a go at the two worksheets accompanying this article.

Further reading:

- Maths and climate change: The melting Arctic
  http://plus.maths.org/issue46/features/wadhams/index.html
- 101 uses of the quadratic equation – a touch of quadratic chaos
  http://plus.maths.org/issue30/features/quadratic/index.html
- An introduction to energy balance models
  http://stratus.astr.ucl.ac.be/textbook/chapter3_node6.xml
- Zero dimensional energy balance models
  http://www.math.nyu.edu/caos_teaching/physical_oceanography/numerical_exercises/ebm/zero_dim_ebm.html